

# An efficient numerical solution for frequency response function of micromechanical resonator arrays<sup>†</sup>

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## Abstract

Array forms of MEMS resonator that uses a specific mid-frequency normal mode have been introduced for acquiring a wider bandwidth of frequency response function (FRF). A conventional frequency response solver based on a modal approach faces computational difficulties in obtaining the FRF of these array forms because of the increase in the order of a linear dynamic model and the number of retained normal modes. The computational difficulties can be resolved by using a substructuring-based model order reduction and a frequency sweep algorithm, which requires a smaller number of retained modes of a reduced dynamic system than the conventional solver. In computing the FRF of a single resonator and its array forms, the presented method shows much better efficiency than the conventional solution by ANSYS as the number of resonators increases. In addition, the effects of multiple resonators in the array forms on filter performance are discussed compared with experimental data.

**Keywords:** Frequency response function; Substructuring-based model order reduction; Frequency sweep algorithm; MEMS ring resonator array; Bandwidth

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## 1. Introduction

Filters are used as frequency selective components in modern wireless communication systems. A filter with a micro-scale mechanical resonator can be integrated on chip; thus it is known to improve insertion loss and short battery life, which are disadvantages of an off-chip component. An important factor in designing a filter is obtaining the desired frequency response functions within a specific range. In particular, a micromechanical filter is typically operated in a range of radio frequency signals and requires a high quality factor, so a lightly damped system operated in a high frequency range should be considered.

Some wireless communication systems are re-

quired to operate at a higher frequency range than others. The higher frequency range can be simply acquired by reducing the geometric scale of a resonator, but the reduction is limited because of manufacturing problems. Without changing the scale, a higher frequency range can be acquired by the use of a mid-frequency normal mode whose eigenvalue is much larger than the smallest eigenvalue. Recently, some researchers have developed a ring resonator with this type of the mode, called the extensional wine glass mode [1]. They found that the mode is apparently identified by comparison with the adjacent modes and the resonance peak is obtained in a higher frequency range without reduction of the scale. Moreover, the ring resonator has been used in array form in order to acquire a frequency response function with a wider bandwidth, which results in a repeated geometry [2]. In this paper, an efficient numerical solution for obtaining the frequency response functions near

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the extensional wine glass mode is presented for a large order numerical model due to the repeated geometry.

A modal approach with finite element modeling has been a famous numerical method in frequency response analysis. The conventional modal approach should require all eigenmodes whose natural frequencies are close to that of the extensional wine glass mode from the lowest one to achieve a desirable accuracy. The higher frequency range acquired by the wine glass mode causes the number of the retained eigenmodes to increase. However, the large number of modes in the large order model causes a dramatic increase in computational cost. Therefore, an efficient method which can reduce the mode number and the model order should be employed to obtain a frequency response function in a higher range.

So far, a substructuring-based model order reduction has been developed to improve the efficiency in a linear dynamic analysis of large-scale systems. Automatic multilevel substructuring [3], substructuring reduction for the iterated improved reduced system [4], fast frequency response analysis [5, 6], and algebraic substructuring [7] are included in this category. A variant of algebraic substructuring among them has recently been developed to solve only the modes of the interior eigenvalues [8], which contribute more to the frequency responses near a specific mode of a higher range than the extreme eigenvalues do. Other numerical techniques that consider low- and high-frequency mode truncations are also required when the interior modes are used. There have been two recently introduced methods to compensate for the low- and high-truncations; one is the frequency sweep algorithm, and the other is the mode acceleration method [9]. The frequency sweep algorithm was initially introduced by Bennighof et al. [3], and its convergence was verified by Ko and Bai when it was applied to compensate for both the truncation errors [10]. A recent research shows that the frequency sweep algorithm has a better convergence rate and is also more robust than the mode acceleration method [11]. Thus, we choose frequency sweep algorithm here.

In this paper, the finite element models of practical resonator arrays were used, and the characteristics of the computed frequency response functions of the resonator arrays were investigated. The frequency response functions of a single resonator were used as referenced results. The stiffness and mass matrices

were constructed by the finite element models, and the damping matrix was approximated to be proportional to the mass and stiffness matrices considering the high quality factor. A general purpose finite element package ANSYS was adopted as a conventional method for comparison with the substructuring-based model order reduction.

## 2. Frequency response analysis by algebraic substructuring

The discretized model of a structure for a continuous single-input and single-output second-order system can be written as

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = bu(t), \quad y(t) = l^T x(t) \quad (1)$$

with the initial conditions  $x(0) = x_0$  and  $\dot{x}(0) = v_0$ . Here,  $t$  is the time variable,  $x(t) \in \mathfrak{R}^N$  is a state vector, and  $N$  is the number of degrees of freedom (DOF).  $u(t)$  is the input excitation force, and  $y(t)$  is the output measurement function.  $b \in \mathfrak{R}^N$  and  $l \in \mathfrak{R}^N$  are the input and output distribution vectors, respectively.  $M, K, D \in \mathfrak{R}^{N \times N}$  are the system mass, stiffness and damping matrices, respectively, where  $D$  is approximated by  $\alpha M + \beta K$ .

For frequency response analysis of  $[\omega_{\min}, \omega_{\max}]$  near a specific mid-frequency mode, the frequency response function of the dynamic system can be given as

$$H(\omega) = l^T [\gamma_1 K^\sigma + \gamma_2 M]^{-1} b, \quad (2)$$

where  $\gamma_1 = \gamma_1(\omega) = 1 + i\omega\beta$ ,  $\gamma_2 = \gamma_2(\omega, \sigma) = -\omega^2 + \sigma + i\omega(\alpha + \sigma\beta)$ , and  $K^\sigma = K - \sigma M$ . The shift is given by  $\sigma = (\omega_{\min}^2 + \omega_{\max}^2)/2$  for retaining a smaller number of the normal modes [8].

In algebraic substructuring (AS) among substructuring-based model order reductions, first the transformation matrix  $L$  is obtained from the shifted eigen-system in the Craig-Bampton form [12], and next the  $S$  matrix, which is composed of  $m$  substructure modes, is obtained. The subspace spanned by the columns of the matrix  $A_m = L^{-1}S$  is called *AS subspace* [10]. Projecting the frequency response function  $H(\omega)$  of (2) onto the AS subspace yields

$$H_m(\omega) = l_m^T [\gamma_1 K_m^\sigma + \gamma_2 M_m]^{-1} b_m = l_m^T G_m^{-1}(\omega) b_m, \quad (3)$$

where  $G_m$  is the dynamic matrix,  $K_m^\sigma = A_m^T K^\sigma A_m$ ,

$M_m = A_m^T M A_m$ ,  $l_m = A_m^T l$ , and  $b_m = A_m^T b$ . Note the difference in Eqs. (2) and (3) that the size of the system matrices is reduced from  $N$  to  $m$ . The value of  $m$  is quite small as compared to  $N$ , and this becomes the primary reason for improving the computational efficiency.

In a large-scale system,  $m$  can be still too large to directly solve the frequency response functions at a large number of frequency sampling points. The computational efficiency can be further improved by employing the mode superposition of the retained normal modes ( $p_n$ ) and the *frequency sweep algorithm*, which was first introduced by Bennighof [3], for the truncated modes ( $p_t$ ). The frequency response function of Eq. (3) is represented by the summation of  $H_n(\omega)$  and  $H_t(\omega)$ , which are computed by  $p_n$  and  $p_t$ , respectively:

$$H_m(\omega) = H_n(\omega) + H_t(\omega) \quad (4)$$

The mode superposition for the projected system on the AS subspace retains  $n$  interior eigenpairs of the eigensystem shifted and projected on the AS subspace and the corresponding eigenvectors  $\Phi_n$  are used as the normal modes. From (3), mode superposition yields

$$H_n(\omega) = l_m^T p_n(\omega) = l_m^T \Phi_n (\gamma_1 \Theta_n^\sigma + \gamma_2 I)^{-1} \Phi_n^T b_m, \quad (5)$$

where  $(\Theta_n^\sigma, \Phi_n)$  are the eigenpairs of the eigensystem of  $(K_m^\sigma, M_m)$ . Note that the dynamic matrix in inverse form is reduced to the diagonal matrix of  $n$  order, which is very small as compared to  $m$ .

$H_n(\omega)$  from the mode superposition generally contains errors due to the truncated modes in the low- and high-frequency ranges because the mid-frequency modes are retained. Therefore, the  $p_t$  should consider both truncated modes, which presents a challenging problem to efficiently compensate for the error. This error can be compensated for by the following frequency sweep iteration, in which the convergence has been mathematically verified [10].

$$H_t(\omega) = l_m^T p_t^\ell(\omega), \quad p_t^\ell(\omega) = p_t^{\ell-1}(\omega) + \frac{1}{\gamma_1} [(K_m^\sigma)^{-1} - \Phi_n (\Theta_n^\sigma)^{-1} \Phi_n^T] r_m^{\ell-1}(\omega) \quad (6)$$

for  $\ell = 1, 2, \dots$ , with the initial guess  $p_t^0(\omega)$ , where  $r_m^{\ell-1}(\omega) = b_m - G_m(\omega)(p_n(\omega) + p_t^{\ell-1}(\omega))$ . Note that  $K_m^\sigma$  is a diagonal matrix and then Eq. (6) does not require an expensive computation per iteration rate.  $p_t^0(\omega_k)$  is

determined by a linear extrapolation of the computed vector at a previous frequency if  $k > 2$ ; otherwise,  $p_t^0(\omega_0) = 0$  and  $p_t^0(\omega_1) = p_t^0(\omega_0)$ . A practical stopping criterion is set to test the relative residual error:  $\|r_m(\omega)/b_m\| < \varepsilon$  for a given tolerance  $\varepsilon$ . The iteration scheme is known to have a very fast convergence rate; thus, the cost for the iterative procedure is expected to be minor until the criterion is satisfied. The iteration (6) guarantees its convergence by satisfying the condition that the *contraction ratio*  $\xi$  is smaller than one when the cutoff values for the eigenvalues of the normal modes are determined by

$$\lambda_{\min}^\sigma = -d_{\max} / \xi \quad \text{and} \quad \lambda_{\max}^\sigma = d_{\max} / \xi, \quad (7)$$

where  $d(\omega, \sigma) = |-\gamma_2 / \gamma_1|$  and  $d_{\max} = \max\{d(\omega_k, \sigma), 1 \leq k \leq n_f\}$ , in which  $n_f$  is the number of sampling frequencies. The number of the retained normal modes whose eigenvalues are within the cutoff values becomes much smaller compared to cases in which corresponding eigenvalues are retained from the smallest eigenvalue when the specific mid-frequency mode is employed. That causes another significant saving in the computational cost. Simply put, the presented method has advantages based on the small number of the retained modes in the reduced linear dynamic system.

### 3. Numerical experiments

The FE simulation of a micro-scale ring resonator is used for designing a high-frequency band-pass filter, e.g., channel-select filters for radio front-ends in wireless communications systems. The main goal of those filters is to acquire a desired frequency response function (FRF) in a specific range. The mechanical vibratory resonators have simple shapes such as cantilever beam, square, circle, ring and so on, and usually employ their smallest normal modes for operation; thus the computation of FRF around the normal modes with the simple shape is trivial in terms of modeling and computational cost. However, a recently introduced ring resonator employs the so-called extensional wineglass mode [1], whose natural frequency operates at a very high-frequency range compared as that of the first normal mode; thus, FRF at the much higher frequency range can be easily achieved without changing the geometry of a resonator, in particular, the size, which is inversely proportional to the acquired natural frequencies. Another

recent approach for acquiring the desired FRF is the use of an array form composed of single resonators, which can obtain a desired bandwidth for specific applications [2, 13]. The FRF of the resonator array depends on the number of assembled resonators, the shape of geometry to connect those resonators and so on; thus the design parameters increase and the geometries become more complex than those of the single resonator. Moreover, the numerical calculation of the FRF at the higher range has been a challenging problem.

In this paper, by introducing recently developed methods [8, 10], we numerically demonstrated the FRF of some 2D resonator arrays [2], which were previously obtained through an experimental approach and investigated the performance of the numerical solutions compared to conventional solutions. The FRF obtained by the array forms are compared with that of the single resonator in terms of filter performance such as maximum peak, bandwidth, and ripple. Here we used the maximum peak instead of the insertion loss, which is a typical performance measure of a filter, because we employed the amplitude ratio of mechanical vibration instead of the S21 transmission in experimental data.

### 3.1 Numerical methods

The presented methods on the AS subspace are implemented based on ASEIG [7]. The multilevel partition is automatically done by METIS [14]. The eigenpairs for the projected eigensystem on the AS subspace are computed by the shift-invert Lanczos method of ARPACK [15] with SuperLU [16]. Below are the methods for the next numerical experiments:

AS+FS: Frequency response analysis using algebraic substructuring with a frequency sweep algorithm.

ANSYS-M [17]: Commercial package using mode superposition with normal modes from Block Lanczos and residual flexibility modes, which is a commonly used method in many other commercial packages. The maximum cutoff value for the normal modes is determined by  $\lambda_{\max} = (1.01\omega_{\max})^2$ .  $\lambda_{\min} = 0$  is used because residual flexibility modes compensate for only the error of truncated high frequency modes [18].

ANSYS-D [17]: Commercial package using a direct frequency response solver for comparison in terms of accuracy.

All numerical experiments were conducted on a platform utilizing a 2.66 GHz Intel Xeon processor for AS+FS. This computer has 4 gigabytes (GB) of physical memory and the operating system is Red Hat Linux. The platform that has two 3.00 GHz Intel Xeon processors with 16 GB of physical memory, and Window XP operating system is used for ANSYS. Indirect comparisons were made because of the different platforms.

## 3.2 Numerical results and discussion

### 3.2.1 Single resonator

The finite element model of a single resonator is constructed by brick elements with its order of 24,960 as shown in Fig. 1(a), and it is partitioned by eight level bisections. The geometry and material properties are taken from the 634.6 MHz resonator of Ref. [1], and the ends of the four beams are set by the clamped boundary condition. From eigenvalue computation, the extensional wine glass mode, whose natural frequency is 638.6 MHz, is shown in Fig. 1(b), and the corresponding eigenvalue of the mode is the 281-th small eigenvalue. The natural frequency of the first normal mode is 3.9 MHz; thus, responses near much higher frequency, 638.6 MHz are easily acquired by using the extensional wine glass mode.

The frequency responses within 637.5–639.5 MHz, the range in which the resonance occurs, are computed at 201 frequencies. The damping coefficients are assumed to be  $\alpha=0$  1/sec. and  $\beta=5.36\times 10^{-14}$  sec., by which the quality factor becomes about 4,650 at 638.6 MHz. In an experimental approach [1], the driving force of the resonator is given at the periphery of quadrant 2, as shown in Fig. 1(a), by the elastostatic compressive force and then quadrants 2 and 4 are harmonically stretched and compressed under the harmonic forces as shown in Fig. 1(b). This harmonic behavior causes a change of current, which is experimentally detected as output power [19]. In our numerical simulation, a constant pressure representing the lateral dielectric driving of [2] is applied to the surface of quadrant 2, as shown in Fig. 1(a), and displacements of three points located at the periphery of quadrant 4 are detected, referring to numerical simulation for the 1D array [13]. As shown in Fig. 1(a), the total amplitude as output is obtained by summing the amplitudes of the positions T, M, and B as  $d_T+d_M+d_B$ , where  $d_T$ ,  $d_M$ , and  $d_B$  are their displacement amplitudes in the radial direction. Here, the

displacements at only three points are used for easy detection, but a more sophisticated detection model will be required to obtain FRF close to experimental data. Other parameters for the frequency sweep algorithm are set to  $\xi = 0.5$  and  $\varepsilon = 10^{-3}$ . A reference response  $H_{ref}$  in Fig. 2 stands for a static response under the same force without damping and inertia effect, and  $H/H_{ref}$  is then equal to  $x/x_{ref}$ , which is known as the amplitude ratio.

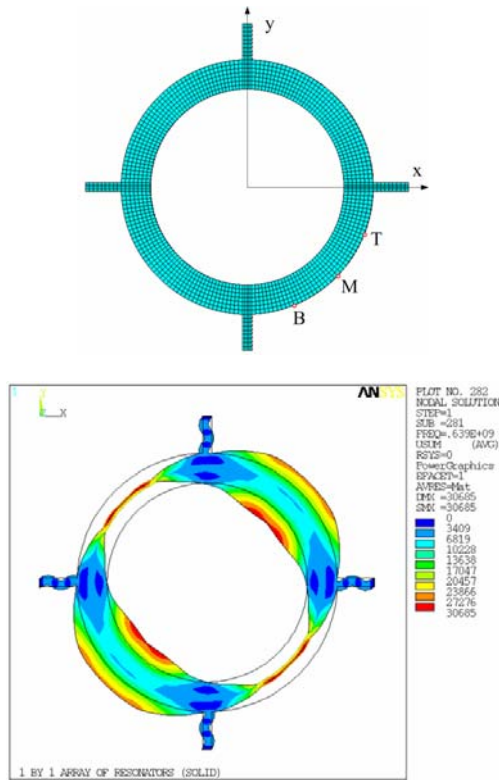


Fig. 1. (a) Finite element model of the single ring resonator and (b) its extensional wine glass mode.

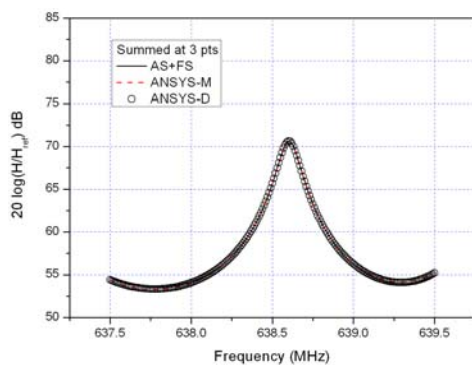


Fig. 2. Frequency response functions of the single resonator.

According to Fig. 2, the frequency response functions of AS+FS and ANSYS-M clearly agree with those of ANSYS-D. The frequency at the resonator peak is located near at 638.6 MHz due to the high quality factor. It is indicated in Table 1 that the time of FS iteration is negligible, and AS+FS uses 5 modes in 469 AS subspace instead of 287 modes in 24,960 FE subspace. Approximately 16 times more time was required for ANSYS-M than for AS+FS because ANSYS-M employed an algorithm that used all low-frequency modes.

3.2.2 Resonator array

Changing the geometric scale of a resonator presents the burden of fulfilling various demands of frequency response functions. A recent approach for obtaining the FRF with a desired bandwidth is the construction of an array form from a single resonator [2, 13]. Here, the influence on the FRF and computational cost are investigated when 2-by-2 and 4-by-4 arrays are used.

A finite element model of a 2-by-2 array with solid elements is illustrated in Fig. 3(a), and its order is 93,720, which is partitioned by 10 level bisections. The vertical beam becomes two times thicker and shorter compared to a single resonator, which is suggested in previous research to obtain a better FRF for the filter design, and beam ends are clamped. Through eigenvalue computations, it was found that the 2-by-2 array has modes close to the extensional wine glass mode around 638 MHz, and the corresponding eigenvalues are located in the vicinity of the 1070-th small eigenvalue; thus, a larger number of normal modes is expected to be retained for FRF calculations than in the single resonator.

Constant pressure is harmonically driven at quadrant 2 of (1, 1) and (2, 1) resonators, and all three points of (1, 2) and (2, 2) resonators are selected for displacement detection; thus, the amplitudes of 6 points are summed into the total amplitude as output.

Table 1. Performance comparison of the single resonator (m: dimension of the AS subspace, n: the number of retained normal modes).

	AS+FS	ANSYS-M
m	469	-
n	5	287
FS time	0.19 sec.	-
Total time	10.34 sec.	162 sec.

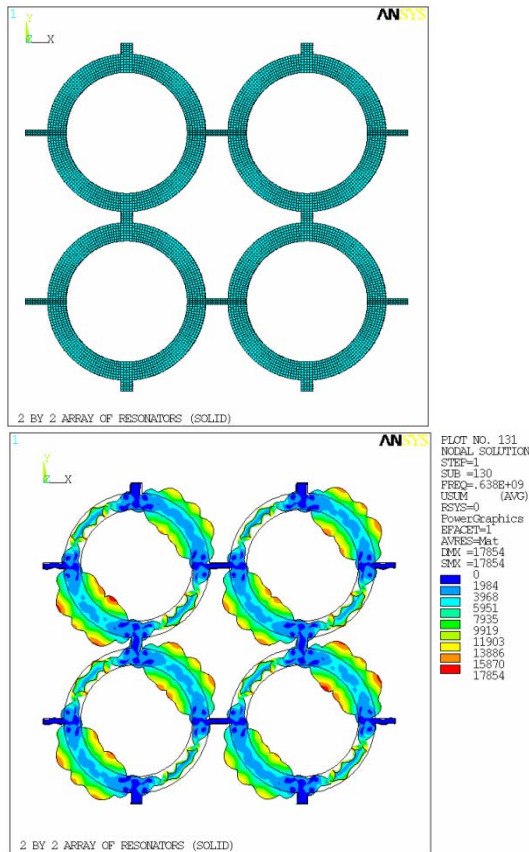


Fig. 3. (a) Finite element model of the 2-by-2 resonator array and (b) a resonant mode close to the extensional wine glass mode.

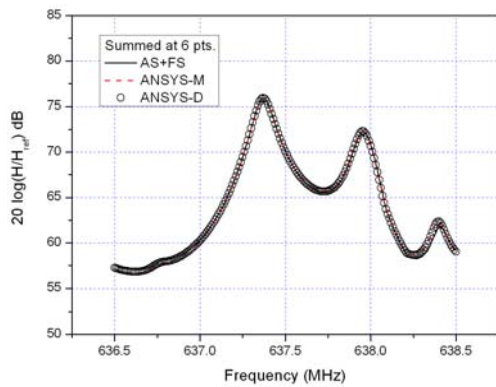


Fig. 4. Frequency response functions of the 2-by-2 resonator array.

The frequency responses within 636.5~638.5 MHz, the range in which the resonance occurs, are computed at 201 frequencies. Other parameters are the same as in the single resonator.

Table 2. Performance comparison of the 2-by-2 resonator array.

	AS+FS	ANSYS-M
m	1,780	-
n	25	1,092
FS time	1.30 sec.	-
Total time	52.76 sec.	2,302 sec.

According to Fig. 4, the frequency response functions of AS+FS and ANSYS-M are in good agreement with those of ANSYS-D. Table 2 shows that the time of FS iteration is minor, and AS+FS uses 25 modes in 1,780 AS subspace instead of 1,092 modes in 93,720 FE subspace. When the ring resonator is used in the array form, ANSYS-M requires more low frequency modes for a larger order model than for the single resonator, which results in much more efficiency in the presented method, that is, 43.6 times more time is required for ANSYS-M than for AS+FS.

A 4-by-4 resonator array is modeled, as shown in Fig. 5(a), and the order of its finite element model is 371,280, which is partitioned by 11 level bisections. Through eigenvalue computations, the 4-by-4 array also has the mode close to the extensional wine glass mode around 638 MHz, and a greater number of eigenmodes is expected to be retained than in the 2x2 array.

Constant pressure is harmonically driven at quadrant 2 of (1, 1), (2, 1), (3, 1), and (4, 1) resonators, and all three points of (1, 4), (2, 4), (3, 4), and (4, 4) resonators are used for displacement detection; thus, the amplitudes of 12 points are summed into the total amplitude as output. The frequency responses within 636.5~638.5 MHz, the range in which the resonance occurs, are computed at 201 frequencies. Other parameters are the same as in the single resonator.

According to Fig. 6, the frequency response functions of AS+FS are in good agreement with those of ANSYS-D. ANSYS-M took 10 hours 38 minutes to compute normal modes, but failed to obtain the frequency response functions due to the shortage of the computer resources. It is indicated in Table 3 that the time of FS iteration is short, and AS+FS uses 90 modes in 7,101 AS subspace instead of 4,335 modes in 371,280 FE subspace. For this case, AS+FS spent less than a hundredth of the elapsed time of ANSYS-M, and ANSYS-D took even less time, 8 hour 46 minutes less than ANSYS-M due to the computation of over 4,335 eigenmodes.

From the frequency response functions of Figs. 2, 4,

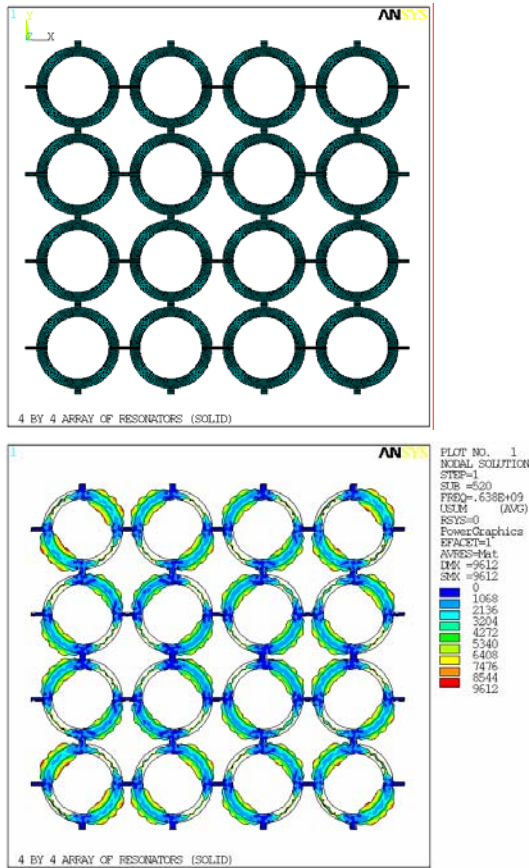


Fig. 5. (a) Finite element model of the 4-by-4 resonator array and (b) a resonant mode close to the extensional wine glass mode.

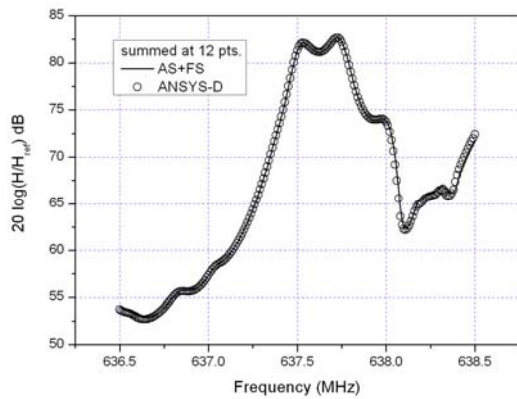


Fig. 6. Frequency response functions of the 4-by-4 resonator array.

and 6, the filter performance data were obtained and then listed in Table 4; ripple is defined from maximum peak to minimum trough. The 3 dB bandwidth

Table 3. Performance comparison of the 4-by-4 resonator array.

	AS+FS	ANSYS-M
M	7,101	-
N	90	4,335
FS time	7.29 sec.	-
Total time	226.49 sec.	> 10 hours

Table 4. Summary of filter performance.

	Single resonator	2-by-2 resonator	4-by-4 resonator
Max. Peak	70.7 dB	76 dB	82.75 dB
Bandwidth	150 KHz	875 KHz	275 KHz
Ripple	-	10.25 dB	1.5 dB

is used for the single resonator, and the bandwidth definition of the Chebyshev filter is used for the array forms because of the existence of the ripple. It should be noticed that the geometry of the resonator array is not exactly the same with that of experimental data [2] because exact information of their dimensions was not available; thus aspect comparisons with experimental data will be discussed.

According to Table 4, the maximum peak improves as the number of resonators increases due to the increase in the number of detection nodes as the insertion loss improves due to increased transduction area in the case of experimental data. The bandwidth of 2-by-2 array becomes much wider as compared to the single resonator, but causes degradation in ripple, namely, larger ripple. Meanwhile, the ripple of the 4-by-4 array improves from 10.25 dB to 1.5 dB over the 2-by-2 array. The bandwidth of the 4-by-4 array becomes shorter than that of the 2-by-2 array, but longer than that of a single resonator. Therefore, the 4-by-4 resonator improves bandwidth at the cost of little ripple, which is similar aspect to the experimental data. Additionally, there is a frequency shift of about 1 MHz in the pass-band of the 4-by-4 resonator related to that of the single resonator, which is also a similar aspect in the experimental data.

As aforementioned, there are many design parameters of an array form to achieve a desired FRF; thus, optimization techniques can be used to obtain a better FRF in future work. In view of computational cost, a conventional method based on a modal approach faces difficulties in obtaining the frequency response function for the array forms with several single resonators due to the dramatic increase in the number of the retained modes and model order, but the sug-

gested method can obtain the FRF with much less cost since fewer retained modes are required for the reduced system.

#### 4. Conclusions

The frequency response functions (FRF) of a ring resonator that uses a specific mid-frequency mode and its array forms were computed and investigated. First, frequency response analyses demonstrated that the resonators in an array form can obtain an FRF with a wider bandwidth at the cost of little ripple than is possible with the single resonators, as suggested in the previously reported experimental results. In obtaining the numerical solutions, the conventional approach with a large number of normal modes was considerably more expensive in terms of computational cost because many eigenvalues are smaller than those of the mid-frequency mode and a larger order model should be constructed as the number of resonators increase in the array forms. However, in the presented method, the mode superposition method is enhanced by the so-called frequency sweep algorithm, which compensates for the errors in low- and high-truncated modes; thus, a smaller number of the retained normal modes are required. Moreover, the order of a linear dynamic model is reduced by projecting on the subspace by algebraic substructuring. Subsequently, the presented method requires a much lower cost for computing the frequency response functions due to both of the reductions in the number of the normal modes and the order of the model. Therefore, the substructuring-based model order reduction and the frequency sweep algorithm can be a competitive analysis tool for the optimal design of a feasible resonator array due to its higher efficiency. The FRFs of the resonator arrays have similar aspects of those of the experimental data, but more sophisticated modeling in a detecting part is required in order to be closer to experimental data. Future work will focus an optimal design for a desired frequency response function of ring resonators in a specific range by appropriate optimization techniques.

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